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ONE FORMULATION OF THE PROBLEM OF ELASTOPLASTIC SEPARATION

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The problem of the beginning of motion of a cut in a plane under symmetric external loading is considered. The material lying on the cut continuation forms a layer (interaction layer). A transition to a plastic state within the layer is assumed to be possible. The behavior of the layer is described by an ideally elastoplastic model, and the plane outside the layer is assumed to be linearly elastic. A system of boundary integral equations for determining the stress–strain state is derived. Based on this system, a discrete model of separation of the layer material is constructed under the assumption of a constant stress–strain state in the element of the interaction layer. The distribution of stresses in the pre-fracture zone is determined.

Key words: *characteristic size, boundary integral equation, linear elasticity, ideally elastoplastic model.*

Two models of formation of material surfaces can be identified in fracture mechanics. The first model, which has been more profoundly studied, is the model of motion of a mathematical cut in a continuous medium [1]. The second model deals with the motion of a physical cut at a certain scale [2, 3]. In most solids, the emergence of new material surfaces is accompanied by the formation of plastic zones in the pre-fracture zone. One of the models with a zone of plastic strains was proposed by Dugdale [4]. In his model, the zone of plastic strains is associated with a narrow band of zero thickness, which is located on the crack continuation. Constant stresses equal to the yield stress of the material act on the band edges. In this case, the Tresca criterion predicts that one mechanism of plastic flow is formally responsible for the plane strain state and the plane stress state, namely, in the coordinate system shown in Fig. 1, the tensile stress equals the yield stress ($\sigma_{11} = 2\tau_s$). Note that available experimental data testify that the shapes of the plastic zone for the plane stress state and the plane strain state are substantially different. The reason is the effect of the stress σ_{22} distributed along the cut axis [5]. According to [5], we have $\sigma_{11} > \sigma_{33} > \sigma_{22}$ for the plane strain state near the crack end and $\sigma_{11} > \sigma_{22} > \sigma_{33} = 0$ for the plane stress state. We also have $\sigma_{11} - \sigma_{22} = 2\tau_s$ in the plane strain state and $\sigma_{11} = 2\tau_s$ in the plane stress state.

In the present paper, the formation of new material surfaces is considered as the motion of a physical cut in a continuous medium. The material lying on the continuation of the physical cut forms the interaction layer. The stress–strain state is assumed to be uniform over the layer thickness [6, 7]. Within the framework of the hypothesis of continuity [6, 7], the interaction-layer thickness δ_0 is assumed to have the minimum possible value. A possibility of the transition of the layer material to the plastic state is implied.

Formulation of the Problem. Let us consider the conditions of the beginning of motion of a physical cut (see Fig. 1). We assume that the material can transfer to a plastic state in some part of the interaction layer ($OSS'O'$ in Fig. 1) of length l_p. In addition to the stress $\sigma_{11}(x_2)$ in the layer, we also take into account the stress $\sigma_{22}(x_2)$ along the cut axis. The relation between the stresses and strains outside the interaction layer is assumed to be described by equations of the linear elasticity theory for the plane strain state ($\varepsilon_{33} = 0$). The behavior of the layer material is considered within the framework of an ideally elastoplastic model [8].

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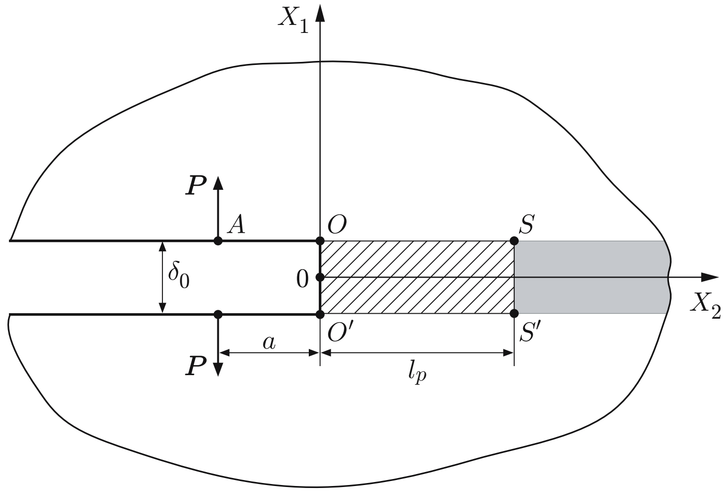

Fig. 1. Scheme of fracture.

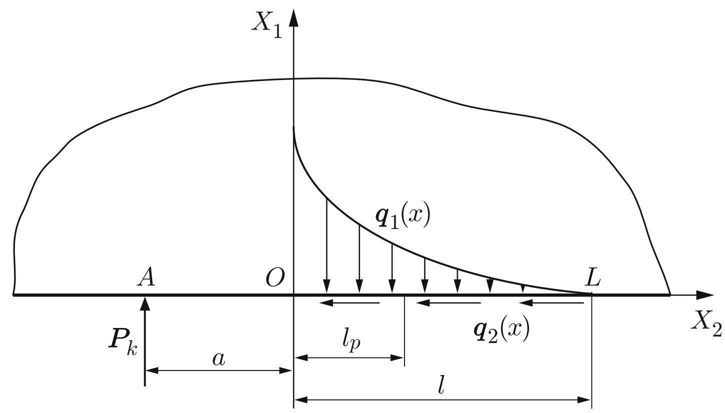

Fig. 2. Distribution of loads on the cut.

The criterion of the transition from the elastic to the plastic state is assumed to be reaching a certain critical value of the maximum shear stress:

$$\max |\sigma_{ii} - \sigma_{jj}| = 2\tau_s \tag{1}$$

(τ_s is the yield stress; $i, j = 1, 2, 3$).

By virtue of symmetry of the problem, we consider only the upper half-plane $x_1 \geqslant \delta_0/2$ (Fig. 2) and replace the action of the layer on the half-plane by the load on this plane:

$$\boldsymbol{q}(x) = -(\hat{\sigma}_{11}\boldsymbol{e}_1 + \hat{\sigma}_{21}\boldsymbol{e}_2).$$

Here $x \equiv x_2/\delta_0$ is the dimensionless coordinate, $\hat{\sigma}_{ij} = \beta\sigma_{ij}$ $(i, j = 1, 2)$ are the dimensionless stresses, $\beta = 2(1-\nu^2)/(\pi E)$ is the material parameter for the Flamant problem [9] in the case of plane strains, E is Young's modulus, and ν is Poisson's ratio.

Flamant's relations [9] relate the external loads $\hat{\sigma}_{11}$ and $\hat{\sigma}_{12}$ to the displacements of the half-plane boundary:

$$\hat{u}_1(x) = -\hat{P} \ln \frac{x+a}{l+a} + \int\limits_0^l \hat{\sigma}_{11}(\xi) \ln \frac{|x-\xi|}{l-\xi}\, d\xi; \tag{2}$$

$$\hat{u}_2(x) = \int\limits_0^l \hat{\sigma}_{12}(\xi) \ln \frac{|x-\xi|}{l-\xi}\, d\xi. \tag{3}$$

Here $\hat{u}_i = u_i/\delta_0$ $(i = 1, 2)$ are the dimensionless displacements, $\hat{P} = P\beta/\delta_0$ is the dimensionless force per unit thickness, and l is the distance from the origin to the point L of zero displacement.

The main postulate of the interaction-layer model [6] is the statement about the uniformity of the stress–strain state over the layer thickness. Therefore, the equilibrium condition implies that

$$\frac{\partial \hat{\sigma}_{22}}{\partial x} = -2\,\hat{\sigma}_{21}. \tag{4}$$

The displacements of the layer boundaries are determined from the conditions

$$\hat{u}_1(x) = \frac{1}{2}\,\varepsilon_{11}(x); \tag{5}$$

$$\hat{u}_2(x) = \int\limits_l^x \varepsilon_{22}(x)\,dx. \tag{6}$$

The stresses (before the yield stress is reached) are related to the strains by Hooke's law

$$\varepsilon_{11} = A\hat{\sigma}_{11} - B\hat{\sigma}_{22}; \tag{7}$$

$$\varepsilon_{22} = A\hat{\sigma}_{22} - B\hat{\sigma}_{11}, \tag{8}$$

where $A = \pi/2$ and $B = \nu\pi/(2(1-\nu))$ are dimensionless constants.

Under the plane strain conditions, we have $\sigma_{11} > \sigma_{33} \geqslant \sigma_{22}$ in the pre-fracture zone [5]. Hence, the Tresca yield criterion (1) for this loading scheme is determined by the expression

$$\hat{\sigma}_{11} - \hat{\sigma}_{22} = 2\hat{\tau}_s, \tag{9}$$

where $\hat{\tau}_s = \beta\tau_s$ is the dimensionless yield stress.

We differentiate Eq. (3) with respect to x:

$$\varepsilon_{22} = \frac{d\hat{u}_2}{dx} = \int\limits_0^l \hat{\sigma}_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi. \tag{10}$$

Taking into account Eq. (5), we write Eq. (2) in the form

$$\varepsilon_{11} = -2\hat{P}\ln\frac{x+a}{l+a} + 2\int\limits_0^l \hat{\sigma}_{11}(\xi)\ln\frac{|x-\xi|}{l-\xi}\,d\xi. \tag{11}$$

As the layer boundary separates the zones of elastic and plastic deformation, relations (10) and (11) determine the total strains of the interaction layer at the stages of both elastic and elastoplastic deformation. We assume that the strains are small and that the following expansion is valid in the case of elastoplastic deformation:

$$\varepsilon_{ii} = \varepsilon_{ii}^e + \varepsilon_{ii}^p, \qquad i = 1, 2.$$

Here ε_{ii}^e and ε_{ii}^p are the elastic and plastic components of the total strain.

Criterion (9) holds at the moment of the transition from the elastic to the plastic state. With allowance for Eqs. (7) and (8), the expressions for the elastic components of the total strain in the layer acquire the form

$$\varepsilon_{11}^e = A\hat{\sigma}_{11}^e - B\hat{\sigma}_{22}^e, \qquad \varepsilon_{22}^e = A\hat{\sigma}_{22}^e - B\hat{\sigma}_{11}^e,$$

where $\hat{\sigma}_{11}^e$ and $\hat{\sigma}_{22}^e$ are the stresses corresponding to the transition to the plastic state.

We assume that the elastic and plastic components of the transverse strain are equal to zero ($\varepsilon_{33}^e = 0$ and $\varepsilon_{33}^p = 0$) and that the material is plastically incompressible:

$$\varepsilon_{11}^p + \varepsilon_{22}^p = 0.$$

Taking into account Eqs. (10) and (11) and also the condition $\varepsilon_{33} = 0$, we determine the change in the volume along the layer owing to the motion of the "walls" bounding its elastic space:

$$\int_0^l \hat\sigma_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi + 2\int_0^l \hat\sigma_{11}(\xi)\ln\frac{|x-\xi|}{l-\xi}\,d\xi - 2\hat P\ln\frac{x+a}{l+a} = \varepsilon_{11}(x)+\varepsilon_{22}(x) = \theta(x). \tag{12}$$

Note that Eq. (12) is valid in the cases of both elastic and elastoplastic deformation of the layer. The law of the layer-volume variation can be presented as

$$\theta(x) = \begin{cases} \varepsilon^e_{11}+\varepsilon^e_{22}+\varepsilon^p_{11}+\varepsilon^p_{22}, & 0\leqslant x\leqslant l_p, \\ \varepsilon_{11}+\varepsilon_{22}, & x>l_p. \end{cases}$$

By virtue of plastic incompressibility, we obtain

$$\theta(x) = \begin{cases} \varepsilon^e_{11}+\varepsilon^e_{22}, & 0\leqslant x\leqslant l_p, \\ \varepsilon_{11}+\varepsilon_{22}, & x>l_p. \end{cases}$$

Using Hooke's law (7), (8), we present the law of volume variation in the form

$$\theta(x) = (A-B)(\hat\sigma_{11}+\hat\sigma_{22}). \tag{13}$$

From the conditions of identical changes of the volume in Eqs. (12) and (13) and also the condition of equilibrium of the layer element (4), we obtain the following system of equations:

$$\begin{gathered} \int_0^l \hat\sigma_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi + 2\int_0^l \hat\sigma_{11}(\xi)\ln\frac{|x-\xi|}{l-\xi}\,d\xi - 2\hat P\ln\frac{x+a}{l+a} = \theta(x), \\ \frac{\partial\hat\sigma_{22}}{\partial x} = -2\hat\sigma_{21}. \end{gathered} \tag{14}$$

In the elastoplastic region on the segment $0\leqslant x\leqslant l_p$, system (14) is supplemented by the Tresca yield condition (9). As a result, the system of integral and differential equations (14) and (9) becomes closed and acquires the form

$$\begin{gathered} \int_0^l \hat\sigma_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi + 2\int_0^l \hat\sigma_{11}(\xi)\ln\frac{|x-\xi|}{l-\xi}\,d\xi - 2\hat P\ln\frac{x+a}{l+a} = \frac{\pi(\hat\tau_s+\hat\sigma_{22}(x))(1-2\nu)}{1-\nu}, \\ \frac{\partial\hat\sigma_{22}}{\partial x} = -2\hat\sigma_{21}, \qquad \hat\sigma_{11}-\hat\sigma_{22} = 2\hat\tau_s. \end{gathered} \tag{15}$$

In the elastic region, where $x>l_p$ and $\hat\sigma_{11}-\hat\sigma_{22}<2\hat\tau_s$, system (14) is supplemented by the condition of equality of the strains ε_{22} along the layer [these strains are calculated from Eq. (10) and directly from Hooke's law (8)]. As a result, the system of resolving equations has the following form:

$$\begin{gathered} \int_0^l \hat\sigma_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi + 2\int_0^l \hat\sigma_{11}(\xi)\ln\frac{|x-\xi|}{l-\xi}\,d\xi - 2\hat P\ln\frac{x+a}{l+a} = \frac{\pi(\hat\sigma_{11}(x)+\hat\sigma_{22}(x))(1-2\nu)}{2(1-\nu)}, \\ \hat A\hat\sigma_{22}-\hat B\hat\sigma_{11} = \int_0^l \hat\sigma_{12}(\xi)\,\frac{1}{x-\xi}\,d\xi, \qquad \frac{\partial\hat\sigma_{22}}{\partial x} = -2\hat\sigma_{12}. \end{gathered} \tag{16}$$

The main unknowns of systems (15) and (16) are the stress-tensor components and the length of the plastic region. The end face of the initial cut is assumed to be free from loading:

$$\hat\sigma_{22}\Big|_{x=0} = 0. \tag{17}$$

Discrete Model of Deformation of the Interaction Layer. Following [10], we assume in solving the problem that the fracture of a solid is a discrete process; therefore, the stress state in an interaction-layer element of length δ_0 or of unit dimensionless length is assumed to be uniform.

To construct the problem solution with the use of the discrete model, we divide the half-plane boundary OL into N unit elements. Each boundary element k with the coordinates ξ_{k-1}, ξ_k, where $k = 1, \ldots, N$, is characterized by constant (averaged over the element) values of the stresses $\sigma_{11}^{(k)}$, $\sigma_{22}^{(k)}$, and $\sigma_{12}^{(k)}$ determined as

$$\sigma_{ij}^{(k)}(x_{(k)}) = \int\limits_{\xi_{k-1}}^{\xi_k} \hat{\sigma}_{ij}(\xi)\,d\xi, \qquad x_{(k)} = \frac{\xi_k + \xi_{k-1}}{2}.$$

As a result, the integrals in the equations of systems (15) and (16) are presented in the form of the corresponding sums. For discretization of the equilibrium equation (4), we integrate it with respect to the kth element. We obtain $\sigma_{22}^{(k)} - \sigma_{22}^{(k-1)} = -2\sigma_{21}^{(k)}$; for the first element, with allowance for Eq. (17), we have $\sigma_{22}^{(1)} = -2\sigma_{21}^{(1)}$. It should be noted that this approach has a certain similarity to the method of boundary elements [9] with constant approximation. The main distinction of the present approach is the fact that division of the half-plane boundary OL into elements of smaller size makes no sense. The value of δ_0 is bounded by the hypothesis of continuity.

Thus, on the basis of systems (15) and (16), we obtain a discrete model of elastoplastic deformation of the interaction layer, where the first l elements are in the plastic state, the $(l+1)$th element transforms to the plastic state, and the remaining elements experience elastic deformation. This model consists of three systems.

1) Equations that describe the plastic region:

$$\begin{gathered}\sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi + 2\sum_{i=1}^{N} \sigma_{11}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \ln\frac{|x_{(i)} - \xi|}{n - \xi}\,d\xi - 2P_{l+1}\ln\frac{x+a}{n+a} = \frac{\pi(\hat{\tau}_s + \hat{\sigma}_{22}^{(k)})(1-2\nu)}{1-\nu}, \\ \sigma_{22}^{(k)} - \sigma_{22}^{(k-1)} = -2\sigma_{21}^{(k)}, \quad \sigma_{11}^{(k)} - \sigma_{22}^{(k)} = 2\hat{\tau}_s, \quad k = 1, \ldots, l, \qquad \sigma_{22}^{(0)} = 0.\end{gathered} \tag{18}$$

2) Equations that describe the transition of the $(l+1)$th element from the elastic to the plastic state:

$$\begin{gathered}\sigma_{11}^{(k)} - \sigma_{22}^{(k)} = 2\hat{\tau}_s, \\ \sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi + 2\sum_{i=1}^{N} \sigma_{11}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \ln\frac{|x_{(i)} - \xi|}{l - \xi}\,d\xi - 2P_{l+1}\ln\frac{x+a}{n+a} = \frac{\pi(\sigma_{11}^{(k)}(x_{(k)}) + \sigma_{22}^{(k)}(x_{(k)}))(1-2\nu)}{2(1-\nu)}, \\ A\sigma_{22}^{(k)}(x_{(k)}) - B\sigma_{11}^{(k)}(x_{(k)}) = \sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi, \\ \sigma_{22}^{(k)} - \sigma_{22}^{(k-1)} = -2\sigma_{21}^{(k)}, \qquad k = l+1.\end{gathered} \tag{19}$$

3) Equations that describe the elastic region:

$$\begin{gathered}\sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi + 2\sum_{i=1}^{N} \sigma_{11}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \ln\frac{|x_{(i)} - \xi|}{l - \xi}\,d\xi - 2P_{l+1}\ln\frac{x+a}{n+a} = \frac{\pi(\sigma_{11}^{(k)}(x_{(k)}) + \sigma_{22}^{(k)}(x_{(k)}))(1-2\nu)}{2(1-\nu)}, \\ A\sigma_{22}^{(k)}(x_{(k)}) - B\sigma_{11}^{(k)}(x_{(k)}) = \sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi, \quad \sigma_{22}^{(k)} - \sigma_{22}^{(k-1)} = -2\sigma_{21}^{(k)}, \qquad k = l+2, \ldots, N.\end{gathered} \tag{20}$$

Note that system (19) contains both the condition of equality of the longitudinal elastic strains

$$A\sigma_{22}^{(l+1)}(x_{(k)}) - B\sigma_{11}^{(l+1)}(x_{(k)}) = \sum_{i=1}^{N} \sigma_{12}^{(i)}(x_{(i)}) \int\limits_{\xi_{i-1}}^{\xi_i} \frac{1}{x_{(i)} - \xi}\,d\xi,$$

and the condition of reaching the plastic state

$$\sigma_{11}^{(l+1)} - \sigma_{22}^{(l+1)} = 2\hat{\tau}_s.$$

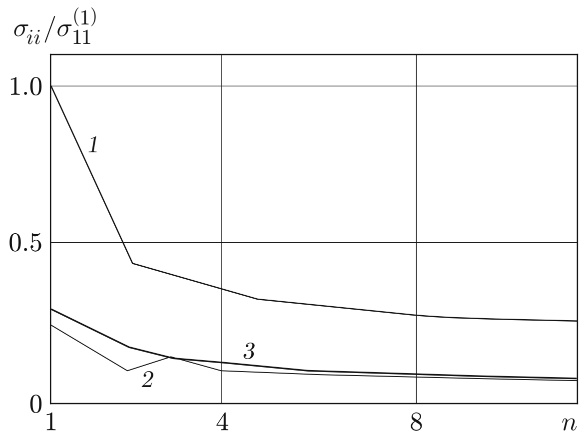

Fig. 3

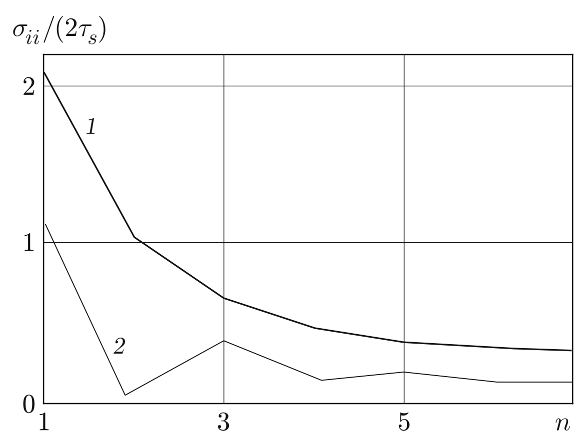

Fig. 4

Fig. 3. Stress distribution on the cut under the load corresponding to the beginning of the yield process in the first element ($n = 1000$, $\nu = 0.25$, and $a = 5$): σ_{11} (1), σ_{22} (2), and σ_{33} (3).

Fig. 4. Stress distribution on the cut under the load corresponding to the beginning of the yield process in the second element ($n = 1000$, $\nu = 0.25$, and $a = 5$): σ_{11} (1) and σ_{22} at $2\tau_s/E = 2 \cdot 10^{-3}$ (2).

In addition, redistribution of stresses in a developed plastic flow was taken into account: if it was found that $\sigma_{22}^{(l)} > \sigma_{33}^{(l)}$ for some element of the medium, then criterion (1) for this element was taken in the form $\sigma_{11}^{(l)} - \sigma_{33}^{(l)} = 2\hat{\tau}_s$.

The full system of discrete deformation consisting of subsystems (18)–(20) contains $3N + 1$ equations. The unknowns are $3N$ generalized stresses and the critical force P_{l+1} that ensures the existence of this stress state.

We intend to solve the resultant system of linear equations with respect to $3N$ generalized stresses and the critical force P_{l+1} by an iterative approach because of significant physical nonlinearity of the problem. In constructing the iterative process for the zone $l + 1$ and the critical load P_{l+1}, we use the known solution for the zone l and the load P_l. In this case, as an iterative parameter, we use the value of the stress σ_{22} in the first equation of system (18) on the first element. In the zero approximation, the stress $\sigma_{22}^{(1)}(P_{l+1})$ in the right side of the first equation of system (18) equals the stress $\sigma_{22}^{(1)}(P_l)$. Let us recall that $\sigma_{22}^{(1)}(P_l)$ is the stress on the first element corresponding to the load P_l. Solving system (18)–(20) under the condition $\sigma_{22}^{(1)}(P_{l+1}) = \sigma_{22}^{(1)}(P_l)$ for the first equation of system (18), we find the distributions of the stresses ${}^{(0)}\sigma_{ij}^k$, $k = 1, \ldots, N$ and the external load ${}^{(0)}P_{l+1}$ in the zero approximation. Based on the found value ${}^{(0)}\sigma_{22}^{(1)}({}^{(0)}P_{l+1})$, we construct the first approximation of the solution of system (18)–(20) under the condition ${}^{(1)}\sigma_{22}^{(1)}(P_{l+1}) = {}^{(0)}\sigma_{22}^{(1)}({}^{(0)}P_{l+1})$ and determine the distributions of the stresses ${}^{(1)}\sigma_{ij}^k$, $k = 1, \ldots, N$ and the external load ${}^{(1)}P_{l+1}$. Subsequent approximations are constructed in the same manner.

Note that the iterations were constructed in the above-described variant with the use of the condition $\sigma_{22}^{(1)}(P_{l+1}) = \sigma_{22}^{(1)}(P_l)$ for one element only. For the remaining k elements, the values of the stresses ${}^{(i)}\sigma_{22}^{(k)}(P_{l+1})$ (i is the iteration number) were found by solving the corresponding system of linear equations. In other words, the distribution of the stresses $\sigma_{22}^{(k)}(P_{l+1})$ in the zero approximation was not assumed to coincide with the distribution of these stresses corresponding to the length of the plasticity zone l. The calculation results show that six iterations at each step are sufficient for the convergence of the iterative process at which the stress field at the jth step differs from the stress field obtained at the $(j - 1)$th step by less than 0.1%. Satisfaction of the corresponding criterion [1–3, 6, 9, 11] in the first element of the layer indicates the beginning of the formation of new material surfaces. The calculations show that the stress state remains almost unchanged if there are more than 1000 elements, which indirectly evidences the calculation convergence.

Figure 3 shows the distributions of the stresses normalized to the stress $\sigma_{11}^{(1)}$ in the first element under the critical load P_1 corresponding to the beginning of the yield process in the first element for $n = 1000$, $\nu = 0.25$, and $a = 5$.

Note that $\sigma_{22} = \sigma_{33} = 0$ for $\nu = 0$, and the inequality $\sigma_{22} < \sigma_{33}$ holds for the remaining admissible values of Poisson's ratio, which confirms the validity of the choice of the yield criterion in the form (9).

Figure 4 shows the distributions of the stresses normalized to the yield stress over the elements at the moment when the yield stress is reached in the second element (load P_2). It is seen that the stresses in a plastically deformed element, which is in the plane strain state, can be substantially higher than the yield stress (see [5]). This can be attributed to making allowance for all stresses in modeling the pre-fracture zone of the elastoplastic material in a plane state that involves hydrostatics with tensile stresses. If the stresses σ_{22} and σ_{33} are neglected by virtue of their small values as compared with σ_{11}, then the stresses in the pre-fracture zone do not exceed the yield stress. Such a situation is observed in the case of the plane stress state.

In determining the stress states corresponding to the transition of the third and second elements to the plastic state, the critical loads are $P_3 = 3.7P_1$ and $P_2 = 1.6P_1$. Hence, an increase in the load necessary for the plastic zone to move in the axial direction under the plane strain condition indicates the possibility of its extension in the X_1 direction (see Fig. 1).

If the stress σ_{11} in the case of irreversible deformation is considered as the action of adhesion forces [12], then these forces in the present formulation of the problem are determined by solving an appropriate boundary-value problem rather than introduced *a priori*.

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